

## Using Applets for Physics Education- Case Study “Nonlinear Systems and Chaos”

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**ABSTRACT:** Simulations with Java applets can be a reasonable tool to assist in physics education, supplementing theoretical lectures and classical experiments. For this purpose the concepts and graphical interfaces of the applets have to be designed according to didactic needs. This understood they make possible to include modern topics into beginner's courses, which usually can not be presented due to their mathematical complexity. This is exemplified with applets supporting a short course on basic phenomena of chaotic systems. They simulate the free and the harmonically excited mathematical pendulum, displaying space-time and phase space diagrams. Further applets allow to study Poincare sections or the dependence on initial conditions. Besides the presentation of qualitative features like basic motion types these applets can be used as “virtual experiments” leading to quantitative results. For each applet its design and function are explained together with its purpose during the course considered here.

### INTRODUCTION

Java applets simulating physical experiments have become quite popular in physics education for several reasons:

- The results of the simulation are free from disturbing influences, they lead to reproducible results.
- The graphical representation of the experimental situation can be reduced to the essential aspects.
- The applets can be easily included in HTML pages for the web or on CD-ROMs.

Of course simulations can't replace real experiments in the classroom or lab: The difficulties, which every experimenter faces, and the care and accuracy necessary to cope with them are an integral part of the experiences students have to make.

An increasing number of new physics textbooks contains such applets on a supplementing CD-ROM [1,2]. The internet is a treasure trove of applets for all branches of physics. Good starting points are f. i. the link collections of PhysicsWeb [3], J. Loviscach [4] or the project „physik multimedial“ [5]. Especially in the area of non-linear oscillations and chaotic systems there exist fine applets [6], which complement and largely extend the examples described below.

Similar approaches use software tools like Matlab [7] or Maple [8]. Unlike the applets presented here, they need substantial mathematical prerequisites, which make them better suited for advanced courses.

### THE COURSE “NONLINEAR SYSTEMS AND CHAOS”

Nonlinear vibrations and chaotic systems have found attention in the engineering sciences as can be seen by a growing number of corresponding textbooks [9,10]. Sometimes they are integrated into standard courses on the theory of vibrations [11], but usually they fall victim to the students' lack of necessary mathematical prerequisites. The goal of the short course described here, which is part of a standard physics course for mechanical engineers [12], is to close this gap and to provide a basic physical understanding of nonlinear and chaotic systems.

For a successful comprehension of the course one needs the following prerequisites: In mathematics

- trigonometry,
- basic vector calculus,
- notion of the derivative

which are taught in beginner's mathematics courses, in physics

- decomposition of forces,
- Newton's equation of motion,
- basics of linear oscillations (spring pendulum),

which have been presented in a previous physics course and in technical mechanics. The concept of the equation of motion as a differential equation is known from physics, further knowledge about ordinary differential equations, especially methods for finding solutions, are not needed here.

At the end of the course the participants should know the basic types of motion of nonlinear systems:

- harmonic oscillation as an approximation for small amplitudes,
- dependency of the frequency on the amplitude,
- period doubling phenomenon,
- apparently irregular, “chaotic” motion.

They should know the phase space diagram as a tool to investigate nonlinear systems and be able to apply it – together with the Poincare cut – to identify chaotic behavior. Finally they should know how to estimate the dependence of the motion on the initial conditions and its consequences for the prediction of chaotic systems.

The mathematical pendulum with harmonic excitation serves as a fundamental example system. Its equation of motion

$$\ddot{\varphi} + \frac{b}{m} \dot{\varphi} + \frac{g}{l} \sin(\varphi) = \frac{\hat{F}}{ml} \cos(\omega_{ext} t) \quad (1)$$

can be derived explicitly using only the stated mathematical prerequisites. It is examined closely with the given applets, alternating between demonstrations by the lecturer and own experiments done by the students. The applets are used to study the qualitative behavior and for concrete “measurements” leading to quantitative relations, f. i. between the oscillation period and the amplitude for the free pendulum. As a complementing example the Duffing oscillator is pre-

sented, which shows a very clear series of period doublings on the route to chaos.

## BASIC STRUCTURE OF THE APPLETS

The applets should be usable without further instructions. For that reason they all have the same basic structure: They consist of an input panel with all input controls, an object panel, which displays the simulated object in a schematic way, and an output panel showing the results of measurements as numbers or graphically. They have a characteristic color and are arranged in a similar layout.

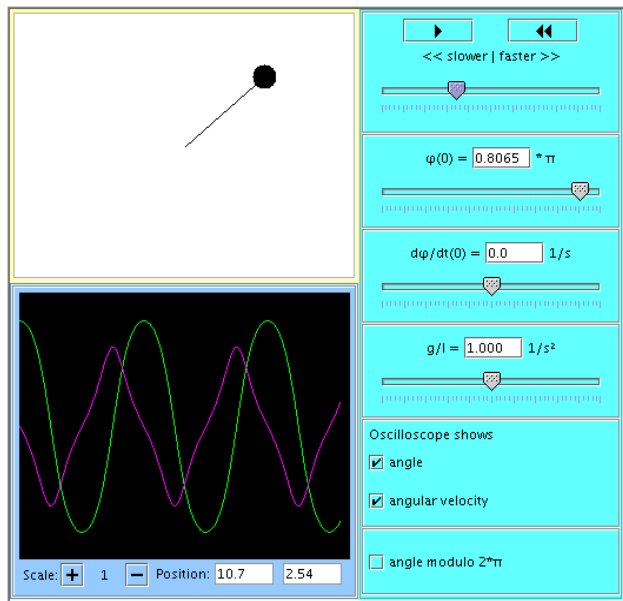


Figure 1 Applet „Mathematical Pendulum“

All manipulations on the simulated system are done via special input elements: a timer, numerical input fields and selection boxes. Other possibilities like dragging of objects with the mouse are ruled out because they are not immediately obvious to the user.

The timer allows to change the simulation speed, to pause, continue and restart the simulation by using buttons with well-known icons. All numerical input elements have a slider for quick qualitative input and a text field for explicit typing-in of an exact number, which is only accepted within the range that is defined by the slider. A short text or symbol explains the physical quantity and its unit. Selection boxes are used to choose the curves that are displayed in an oscilloscope. Sometimes they define additional plotting parameters. These boxes replace a legend: Simply switching on and off a box makes evident which curve corresponds to which physical quantity.

The output area mainly consists of a simulated oscilloscope that either shows physically interesting quantities as functions of time or one quantity depending on another one (x-y oscilloscope). Since the magnitudes of the shown parameters can vary largely, the oscilloscope has buttons to increase or decrease the scale in several steps, each time by a factor of two.

Quantitative measurements on the curves can be done by clicking at an arbitrary point in the oscilloscope window. The corresponding coordinates are now displayed, using the proper SI base units, independently of the chosen display scale. To

point out this feature, the cursor changes to a crosshair symbol when the mouse is over the display area of the oscilloscope. To make it possible that several physical quantities can be displayed within a common numerical scale (assuming base units), the initial values of all input parameters are tuned to guarantee that the corresponding physical quantities have similar magnitudes in SI units. Changing system parameters can lead to diverging magnitudes of some quantities. In this case a different scale has to be used for each curve to allow for measurements of comparable accuracy.

In the example applets the object panel shows the motion of the mathematical pendulum in schematic form. A more realistic representation – using three-dimensional or lightning effects - might be desirable, but the necessary programming effort is much larger than the didactic benefit. Moreover the simple animation always reminds of the virtual character of the “experiment”. Some of the applets have no object panel, because it would lead to no new insights and the space is needed for the output area. Sometimes the oscilloscope is assigned to the object panel (by its background color). In this case the displayed curves are – taking a more abstract view point - the proper object of measurements.

## DESCRIPTION AND USAGE OF SELECTED APPLETS

In this section we will present four applets that have been used extensively during the course. Besides the description of the applets the focus will be on their applicability and the corresponding teaching goals.

### Mathematical pendulum

This applets simulates the mathematical pendulum without friction and driving force. Input parameters are the initial conditions for the angle  $\varphi$  and the angular velocity  $\dot{\varphi}$  and the only system parameter  $g/l$ . The oscilloscope shows the time functions of angle and angular velocity, with the additional possibility to reduce the angle to a basic interval of  $-\pi$  to  $\pi$ .

As a first step the applet is used with small initial angles to remind of the basic features of linear oscillators. Then the different kinds of movement for larger initial values are demonstrated. The increasing influence of the non-linearity can best be seen by looking at the angular velocity, while the graph of the angle looks rather “harmonic” for a while. Reaching initial angles near  $180^\circ$  the dependency of the period on the amplitude becomes obvious. Adding an initial angular velocity leads to a looping movement, which can be the starting point of a discussion, whether it is sensible to use angles larger than  $360^\circ$ .

Finally the students experiment with the applet to measure the relationship between the period and the amplitude and display their results graphically. If time and the mathematical skills of the students allow, the instructor can derive the approximation formula

$$T(\varphi_0) \approx T_0 \left( 1 + \frac{1}{16} \varphi_0^2 \right) \quad (2)$$

using the method of harmonic balance [9], and compare it with the experimental results.

## Driven pendulum in phase space

The next applet simulates the mathematical pendulum with harmonic driving force, as given by the following equation of motion (in dimensionally reduced form):

$$\ddot{\varphi} + \lambda \dot{\varphi} + \sin(\varphi) = A \cos(\omega_{ext} t) \quad (3)$$

It contains the usual input controls for the initial conditions and the three parameters  $\lambda$ ,  $A$  and  $\omega_{ext}$ . The x-y-oscilloscope shows the angular velocity over the angle (phase space), where the angle can again be reduced modulo  $2\pi$ . In the object panel the driving force is represented by an arrow that is attached to the pendulum mass. Length and direction of the arrow show the momentary value of the force.

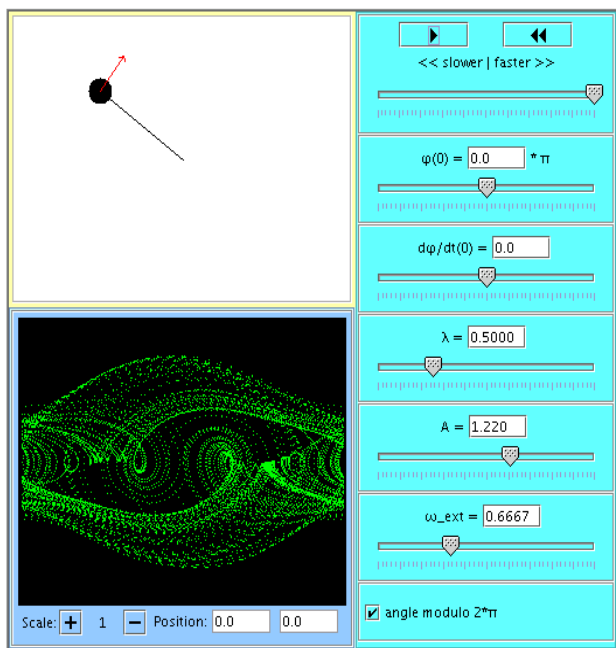


Figure 2 Applet „Driven Pendulum in Phase Space“

During the course the notion of phase space and the driven pendulum have already been introduced. Its behavior has been studied in the time domain, where the analysis is difficult. To understand the complex motion, the two parameters  $\omega_{ext}$  and  $\lambda$  are fixed in the following (useful values being  $\lambda=0.5$ ,  $\omega_{ext}=0.6667$ ) and the results of changing the amplitude  $A$  are demonstrated.

Simple oscillations at  $A = 1.0$  lead to an attractor in the form of a “deformed ellipse”. For  $A = 1.07$  the attractor splits into two overlapping curves, which is a clear sign of the period doubling phenomenon. With  $A = 1.22$  one has reached the chaotic regime. To analyze its behavior one starts with a small scaling factor of the oscilloscope and without reduction of the angle. This clearly shows the random movement between regions with a different number of completed turns. Repeating the experiment with an angle that is reduced to  $2\pi$  and a normal scale one gets the well-known “strange attractor”. In a guided discussion the students now try to find the principal difference between this attractor and the non-chaotic phase portraits. The instructor summarizes the discussion and introduces the notion of “non-integral dimension”, without giving a precise definition. This could be done in a later lecture concentrating on fractals.

For the following “experiments” each group of students gets their own values of  $\lambda$  and  $\omega_{ext}$  with the task to find the type of motion for varying values of  $A$  and to look at the dependence on the initial conditions. The final compilation of results again illustrates the complexity of the chaotic system.

## Poincare section

If only those points are shown in the phase diagram that correspond to a given fixed phase angle  $\alpha$  of the harmonic excitation, a subset of the complete phase portrait is shown that is called a Poincare section. The next applet allows to enter the three system parameters and the phase angle  $\alpha$  and displays the resulting phase image in an x-y oscilloscope occupying the object panel. The interpretation of the result is made difficult by the first points which describe the transient oscillations before the attractor is (approximately) reached. To get rid of these points the applet allows to enter a delay time during which they are suppressed. For larger delay times this leads to a starting phase during which apparently nothing happens. Therefore the current simulated time is displayed in the output panel so that the user can easily estimate when the first points will appear.

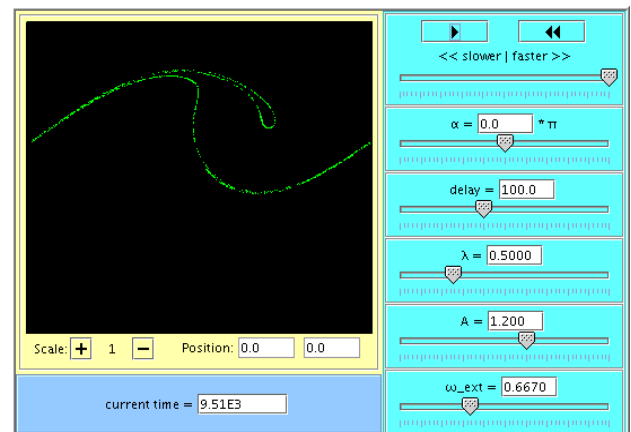


Figure 3 Applet „Poincare Section“

After a short explanation of the function of this applet the students discuss how the basic motion types will appear as Poincare sections. Subsequently the results are demonstrated for the standard values of the parameters: A simple oscillation leads to a single point, period doubling to two points, further doublings to four etc. The strange attractor shows up in the Poincare section as a pattern of many points with interspersed small line parts. Again this brings up the question of the dimension of the curve.

The students now check the results of their previous experiments with this applet. The choice of the delay time needs special care, it has to be large enough to skip the transient phase, especially for small values of the friction coefficient  $\lambda$ . Another point to study is the influence of the phase angle  $\alpha$ . After some tests it should be clear that the complete image of the attractor is the combination of all Poincare sections for the different values of  $\alpha$ .

## Dependence on the initial conditions

The final topic of this course are the consequences of chaotic behavior for the ability to make predictions. The corresponding applet simulates two identical driven pendulums, which start with slightly different initial conditions. Its input values are the

three system parameters and the two initial angles, it displays the angles of both pendulums and their difference as functions of time. The oscilloscope occupies the whole object panel, it is rather large to increase the accuracy which is principally limited by the pixel size.

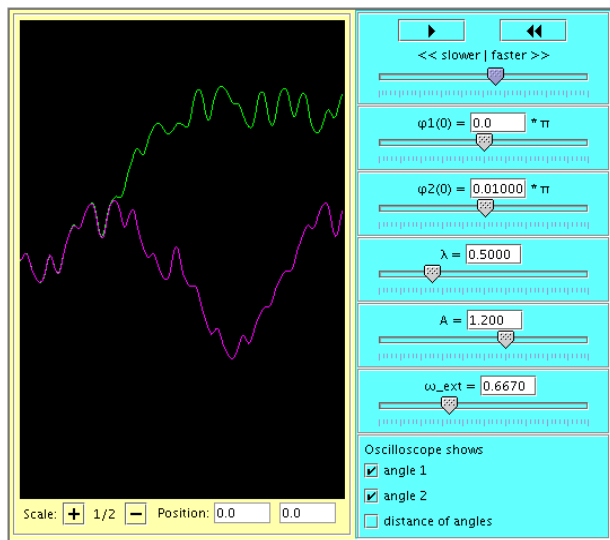


Figure 4 Applet „Initial Conditions“

To get a qualitative overview of the behavior the students start by studying the divergence of the curves for different types of motion and different initial conditions. They get the following results:

- For a simple periodic oscillation the two curves converge (at least modulo  $2\pi$ ) even for very different initial conditions
- If the oscillations have a doubled period, the pendulums can have different motions, but the difference remains small after a transient period. Making the difference of the initial conditions small enough again results in synchronous motion.
- In the chaotic regime both curves diverge strongly after a certain time. For smaller difference of the initial conditions this time grows, but the divergence phenomenon remains.

These observations are the starting point for the introduction of the Lyapunov exponent and the prediction horizon making precise the intuitive notion that chaotic systems defy long-time predictions. Using the applet they can finally by measured approximately for the driven pendulum.

## CONCLUSIONS

Using applets for the presented course has several advantages:

- Listening to the lecturer is supplemented by own activity.
- Quantitative relations are found using experimental methods instead of formal theoretical reasoning.
- The students play an active part in finding the results, they are working as researchers.
- A current physics topic can be presented, which arises the student's curiosity.

The basic problems are the huge amount of work and the profound Java programming knowledge needed to create such applets. This task could be simplified substantially by using a class library designed for physics simulation [13]. To make the most out of this effort one should use the internet to provide

categorized and commented collections of applets that are easy to download and use in physics courses [5]

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